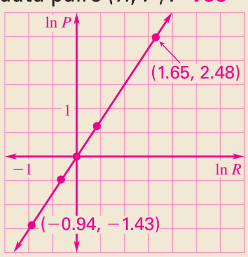
Extra Example 5

The period of a planet is the length of time it takes for the planet to make one complete revolution around the Sun. The table shows the mean distance *R*, in astronomical units, of each of several planets from the Sun and the period *P*, in years, of each of these planets.

Planet	*R*	*P*
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.20	11.9

- Draw a scatter plot of the data pairs ($\ln R$, $\ln P$). Is an exponential model a good fit for the original data pairs (R, P)? **Yes**



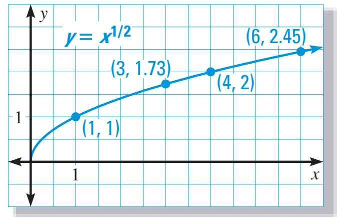
- Find a power model for the original data. **$P = 0.99R^{1.51}$ (Answers may vary slightly depending on the points used.)**

Key Question to Ask for Example 5

- What is the difference, in terms of constants and variables, between an exponential model and a power model? **In an exponential model, the base is a constant and the exponent is a variable, while in a power model, the base is a variable and the exponent is a constant.**

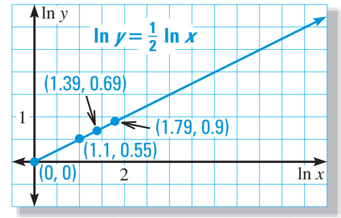
TRANSFORMING POWER DATA A set of more than two points (x, y) fits a power pattern if and only if the set of transformed points $(\ln x, \ln y)$ fits a linear pattern.

Graph of points (x, y)



The graph is a power curve.

Graph of points ($\ln x$, $\ln y$)



The graph is a line.

EXAMPLE 5 Find a power model

BIOLOGY The table at the right shows the typical wingspans x (in feet) and the typical weights y (in pounds) for several types of birds.

- Draw a scatter plot of the data pairs $(\ln x, \ln y)$. Is a power model a good fit for the original data pairs (x, y)?
- Find a power model for the original data.

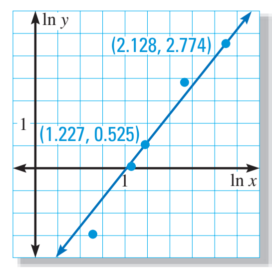
Bird	Wingspan (ft), x	Weight (lb), y
Cuckoo	1.90	0.23
Crow	2.92	1.04
Curlew	3.41	1.69
Goose	5.35	6.76
Vulture	8.40	16.03

Solution

STEP 1 **Use** a calculator to create a table of data pairs $(\ln x, \ln y)$.

$\ln x$	0.642	1.072	1.227	1.677	2.128
$\ln y$	−1.470	0.039	0.525	1.911	2.774

STEP 2 **Plot** the new points as shown. The points lie close to a line, so a power model should be a good fit for the original data.



STEP 3 **Find** a power model $y = ax^b$ by choosing two points on the line, such as (1.227, 0.525) and (2.128, 2.774). Use these points to write an equation of the line. Then solve for y.

$\ln y - y_1 = m(\ln x - x_1)$ — **Equation when axes are $\ln x$ and $\ln y$**

$\ln y - 2.774 = 2.5(\ln x - 2.128)$ — **Substitute.**

$\ln y = 2.5 \ln x - 2.546$ — **Simplify.**

$\ln y = \ln x^{2.5} - 2.546$ — **Power property of logarithms**

$y = e^{\ln x^{2.5} - 2.546}$ — **Exponentiate each side using base e.**

$y = e^{-2.546} \cdot e^{\ln x^{2.5}}$ — **Product of powers property**

$y = 0.0784x^{2.5}$ — **Simplify.**

USE POINT-SLOPE FORM

The slope of the line is $\frac{2.774 - 0.525}{2.128 - 1.227} \approx 2.50$.

Differentiated Instruction

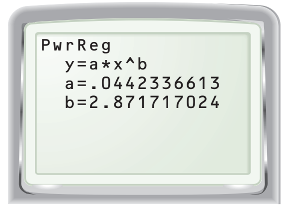
Inclusion For real-world problems like the one in **Example 5**, suggest that students write the data from a graphing calculator in a table to organize their work. They can create either rows or columns, with headers x, $\ln x$, y, and $\ln y$. This will help them see relationships and facilitate their comparison of the values in order to properly determine whether a set of points fits a power model. See also the *Algebra 2 Toolkit* for more strategies.

POWER REGRESSION A graphing calculator that performs power regression uses all of the original data to find the best-fitting model.

EXAMPLE 6 Use power regression

BIOLOGY Use a graphing calculator to find a power model for the data in Example 5. Estimate the weight of a bird with a wingspan of 4.5 feet.

Solution



Enter the original data into a graphing calculator and perform a power regression. The model is $y = 0.0442x^{2.87}$.

Substituting $x = 4.5$ into the model gives $y = 0.0442(4.5)^{2.87} \approx 3.31$ pounds.

✓ GUIDED PRACTICE for Examples 5 and 6

9. The table below shows the atomic number x and the melting point y (in degrees Celsius) for the alkali metals. Find a power model for the data. $y = 397.61x^{-0.639}$

Alkali metal	Lithium	Sodium	Potassium	Rubidium	Cesium
Atomic number, x	3	11	19	37	55
Melting point, y	180.5	97.8	63.7	38.9	28.5

7.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 23, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 27, 33, and 35

SKILL PRACTICE

A

1. VOCABULARY Copy and complete: Given a set of more than two data pairs (x, y), you can decide whether a(n) _?_ function fits the data well by making a scatter plot of the points $(x, \ln y)$. **exponential**

2. ★ WRITING *Explain* how you can determine whether a power function is a good model for a set of data pairs (x, y). **Create a table of the points $(\ln x, \ln y)$. Plot the points. If the points lie close to a line, a power model is a good fit.**

EXAMPLE 1 on p. 529 for Exs. 3–10

WRITING EXPONENTIAL FUNCTIONS Write an exponential function $y = ab^x$ whose graph passes through the given points.

3. (1, 3), (2, 12) $y = \frac{3}{4} \cdot 4^x$
4. (2, 24), (3, 144) $y = \frac{2}{3} \cdot 6^x$
5. (3, 1), (5, 4) $y = \frac{1}{8} \cdot 2^x$
6. (3, 27), (5, 243) $y = 3^x$
7. (1, 2), (3, 50) $y = \frac{2}{5} \cdot 5^x$
8. (1, 40), (3, 640) $y = 10 \cdot 4^x$
9. (−1, 10), (4, 0.31) $y = 4.99 \cdot 0.499^x$
10. (2, 6.4), (5, 409.6) $y = 0.4 \cdot 4^x$

EXAMPLE 2 on p. 530 for Exs. 11–14

FINDING EXPONENTIAL MODELS Use the points (x, y) to draw a scatter plot of the points $(x, \ln y)$. Then find an exponential model for the data. **11–14. See margin for art.**

11. (1, 18), (2, 36), (3, 72), (4, 144), (5, 288) $y = 9(2)^x$
12. (1, 3.3), (2, 10.1), (3, 30.6), (4, 92.7), (5, 280.9) $y = 1.09(3.04)^x$
13. (1, 9.8), (2, 12.2), (3, 15.2), (4, 19), (5, 23.8) $y = 7.83(1.25)^x$
14. (1, 1.4), (2, 6.7), (3, 32.9), (4. 161.4), (5, 790.9) $y = 0.284(4.88)^x$

Extra Example 6

Use a graphing calculator to find a power model for the data in Extra Example 5. Estimate the period of Saturn, whose mean distance from the Sun is 9.54 astronomical units. **$P = R^{1.5}$; about 29.1 yr**

Key Question to Ask for Example 6

- If the methods illustrated in Examples 5 and 6 for finding the equation of the best-fitting power model give different results, which equation do you think would be most accurate? Explain. **The power regression method in Example 6 is generally more accurate because it uses all of the original data points, rather than just two points.**

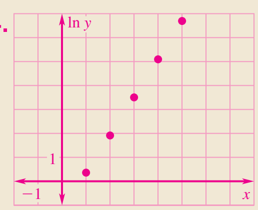
Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you determine whether a set of data fits an exponential pattern or a power pattern?

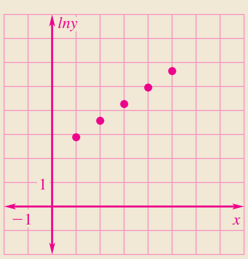
- **Two points determine an exponential model or a power model.**
- **The equation of an exponential or power model can be found from two points on the graph by solving a nonlinear system of equations.**

If the set of transformed points $(x, \ln y)$ fits a linear pattern, then the set of original points (x, y) fits an exponential pattern. If the set of transformed points $(\ln x, \ln y)$ fits a linear pattern, then the set of original points (x, y) fits a power pattern.

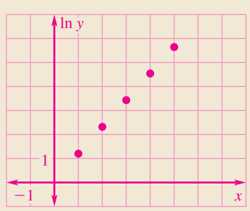
14.



11.



12.



13.